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SPACE TRANSPORTATION VEHICLE DESIGN EVALUATION USING SATURATED DESIGNS

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An important objective in the preliminary design and evaluation of space transportation vehicles is to find the best values of design variables that optimize the performance characteristic (e.g. dry weight). For a given configuration, the vehicle performance can be determined by the use of complex sizing and performance evaluation computer programs. These complex computer programs utilize iterative algorithms and they are generally too expensive and/or difficult to use directly in multidisciplinary design optimization. An alternative is to use response surface methodology (RSM) and obtain quadratic polynomial approximations to the functional relationships between performance characteristics and design variables [2]. In RSM, these approximation models are then used to determine optimum design parameter values and for rapid sensitivity studies.

Constructing a second-order model requires that "n" design parameters be studied at least at 3 levels (values) so that the coefficients in the model can be estimated. Therefore, 3^n factorial experiments (point designs or observations) may be necessary. For small values of "n" such as two or three, this design works well. However, when a large number of design parameters are under study, the number of design points required for a full-factorial design may become excessive.

Fortunately, these quadratic polynomial approximations can be obtained by selecting an efficient design matrix using central composite designs (CCD) from design of experiments theory [3,7]. Each unique point design from the CCD matrix is then conducted using computerized analysis tools (e.g. POST, CONSIZ, etc.). In the next step, least squares regression analysis is used to calculate the quadratic polynomial coefficients from the data.

However, in some multidisciplinary applications involving a large number of design variables and several disciplines, the computerized performance synthesis programs may get too time consuming and expensive to run even with the use of efficient central composite designs. In such cases, it may be preferable to keep the number of design points to an absolute minimum and trade some model accuracy with cost. For this purpose, another class of experimental designs, called saturated D-optimal designs may be utilized for generating a matrix of vehicle designs. A design is called saturated when the number of design points is exactly equal to the number of terms in the model to be fitted [4,5]. As a result, saturated designs require the absolute minimum number of design points $((n+1)(n+2)/2)$ to estimate the quadratic polynomial model coefficients.

Saturated designs can be generated using the D-optimality criterion. A good saturated design should give rise to least squares estimates with minimum generalized variance. To achieve this, the determinant $|X'X|$, which is a measure of variance, should be as large as possible [1,6]. Designs that maximize $|X'X|$ are referred to as D-optimal designs.

D-optimal designs are constructed by selecting an optimal set of points from a candidate set that one wish to consider. The D-optimal design is made up of points from the candidate set using an exchange algorithm to maximize the determinant of $X'X$. However, this is a very difficult problem due to the large number of variables which must be handled (np) and due to the complicated nature of the objective function X'X, especially the existence of several local maxima [1,4].

A number of authors developed algorithms and obtained D-optimal designs for specific models using mathematical programming methods to optimize the determinant |X'X| [1,3,6,9]. Box and Draper list several saturated quadratic designs for "n" variables and suggest optimal settings of the design points by using the D-optimality criterion of maximizing the determinant |X'X| for the cases of n=2 and 3 variables [1,3]. Box and Draper generalized these designs for $n \geq 4$ [1]. There is some argument in the literature about the D-optimality of these generalized designs when $n \geq 4$, in the sense of maximizing |X'X| [3,4,5]. Box and Draper pointed out that Dubova and Federov had found a better design for n=4 [1,3]. However, Box and Draper argue that their generalized designs provide saturated designs with reasonably high D-efficiency when the number of variables exceeds four [1,3] and these designs should work well.

On the limiting side, D-optimal designs do not contain an equal number of high-level, medium-level and low-level values. Classical designs like, orthogonal arrays and central composite designs are based on balanced and orthogonal mathematical patterns, while D-optimal designs are straight optimizations of a criterion based on the model to be fit [10]. There appears to be no guarantee that the result is actually optimal since the search for D-optimal designs is fraught with local and multiple optima problems [4]. However, these designs appear to work reasonably well in initial screening situations in which it is expected that there will only be a few important parameters [10]. As a result, saturated D-optimal designs may be used in lieu of central composite designs when there are large number of variables under study and experimental point design effort is very expensive.

References:

1. Box, M.J. and Draper, R.D., 1974, "On-Mimimum Point Second-Order Designs," Technometrics, Vol. 16, No. 4, November.
2. Craig, J.A., 1978, "D-Optimal Design Method: Final Report and User's Manual," USAF Contract F33615-78-C-3011, FZM-6777, General Dynamics, Forty Worth Division.
3. Khuri, A.I., and Cornell, J.A., 1987, Response Surfaces: Designs and Analyses, Marcel Dekker Inc., New York.
4. Lucas, J.M., 1974, "Optimum Composite Designs," Technometrics, Vol. 16, No. 4, November.
5. Mitchell, T.J., 1974, "Computer Construction of ' D-Optimal' First Order Designs," Technometrics, Vol. 16, No. 2, May.
6. Mitchell, T.J., 1974, "An Algorithm for the Construction of ' D-Optimal' Experimental Designs," Technometrics, Vol. 16, No. 2, May.
7. Myers, R.H., 1971, Response Surface Methodology, Virginia Commonwealth University, Allyn and Bacon Inc., Boston Mass.
8. Schnackel, J. and Dovenmuehle, R., 1990, "Statistical Experimental Design and its Role in Aerospace Vehicle Design Efforts," AIAA-90-1809, February.
9. St. John, R.C., 1975, "D-Optimality for Regression Designs: A Review," Technometrics, Vol. 17, No. 1, February.
10. D.K. Lin, 1993, "Another Look at First-Order Saturated Designs: The p-efficient Designs," Technometrics, Vol. 35, No. 3, August .